Heterogeneity and Load Balance in Distributed Hash Tables

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Joint Work with Alex Fabrikant and Ion Stoica

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The goals

- Distributed Hash Tables partition an ID space among n nodes
  - Typically: each node picks one random ID
  - Node owns region between its predecessor and its own ID
  - Some nodes get $\log n$ times their fair share of ID space
- **Goal 1:** Fair partitioning of ID space
  - If load distributed uniformly in ID space, then fair partitioning $\Rightarrow$ load balanced system
- **Goal 2:** Fair partitioning when node capacities are heterogeneous
- **Goal 3:** Use heterogeneity to our advantage to reduce route length in overlay that connects nodes

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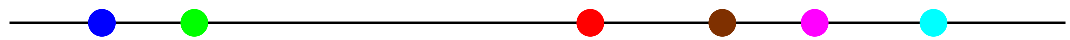

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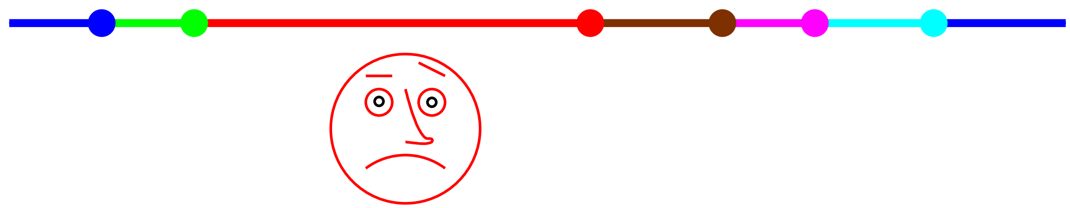

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Model & performance metric

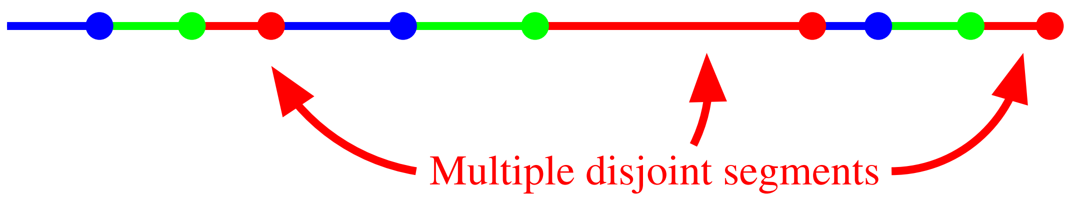
- n nodes
- Each node v has a capacity c_v (e.g. bandwidth)
- Average capacity is 1, total capacity n
- *Share* of node v is

$$\mathrm{share}(v) = \frac{\text{fraction of ID space that } v \text{ owns}}{c_v / n}.$$

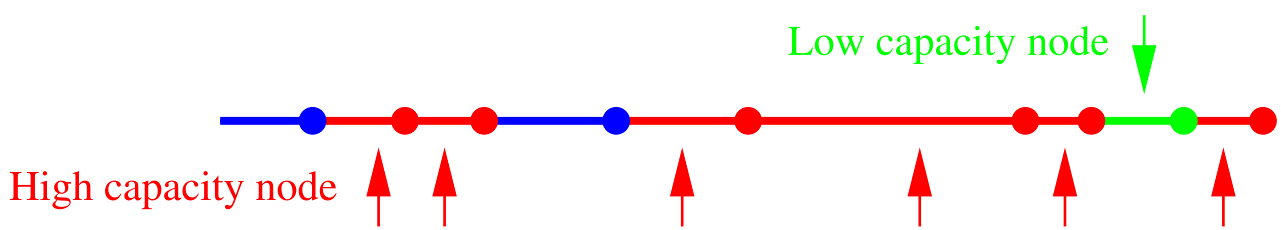
- Want low maximum share
- Perfect partitioning has max. share = 1.

The virtual server solution

- **Goal 1: Load balance**
  - Each node picks $\Theta(\log n)$ IDs (like simulating $\Theta(\log n)$ nodes)
  - Maximum share is $O(1)$ with high probability (w.h.p.) in homogeneous system

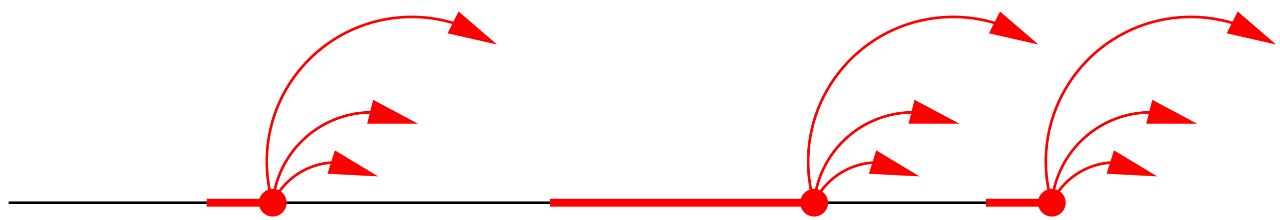


- **Goal 2: Handle heterogeneity**
  - Node of capacity c simulates $\Theta(c \log n)$ nodes
  - Maximum share is $O(1)$ w.h.p. for any capacity distribution



Problems

- To route between nodes, construct an *overlay network*
- With $\Theta(\log n)$ IDs, must maintain $\Theta(\log n)$ times as many overlay connections!



- Other proposals use one ID per node, but...
  - all require reassignment of IDs in response to churn, and load movement is costly
  - none handles heterogeneity directly
  - some can't compute node IDs as hash of IP address for security
  - some are limited in the achievable quality of load balance
  - some are complicated

Our Approach

- Our solution: Low Cost Virtual Servers
- Pick $\Theta(c_v \log n)$ IDs for node of capacity c_v as before...
- ...but *cluster them* in a random fraction $\Theta(\frac{c_v \log n}{n})$ of the ID space
  - Random starting location r
  - Pick $\Theta(c_v \log n)$ IDs spaced at intervals of $\approx \frac{1}{n}$ (with random perturbation)

- Ownership of ID space is still in disjoint segments
- Why does this help?

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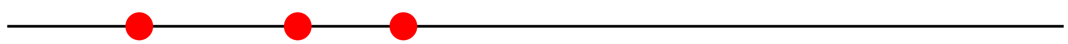

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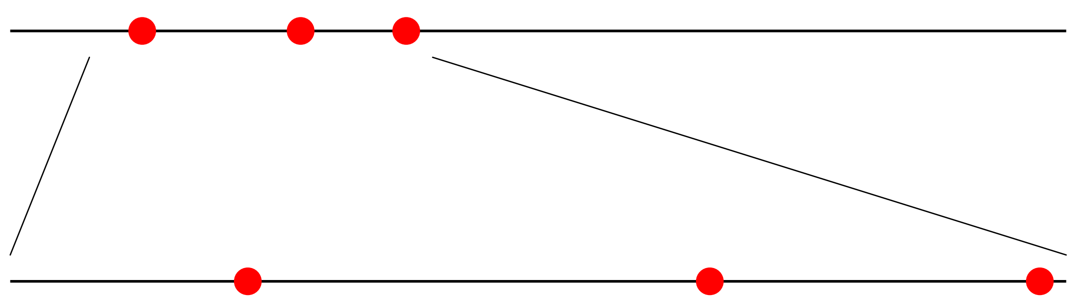

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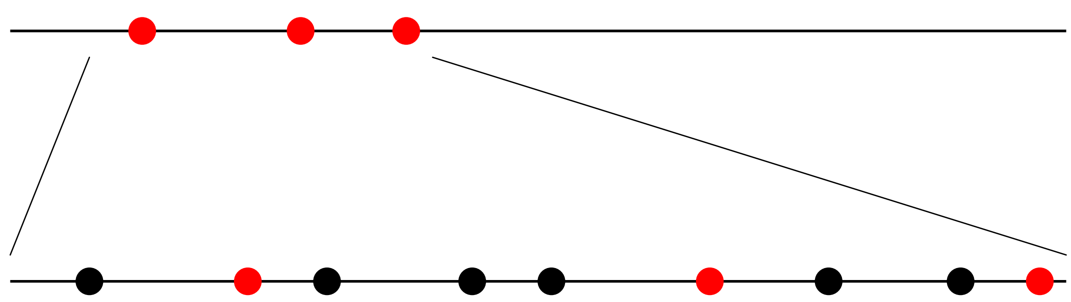

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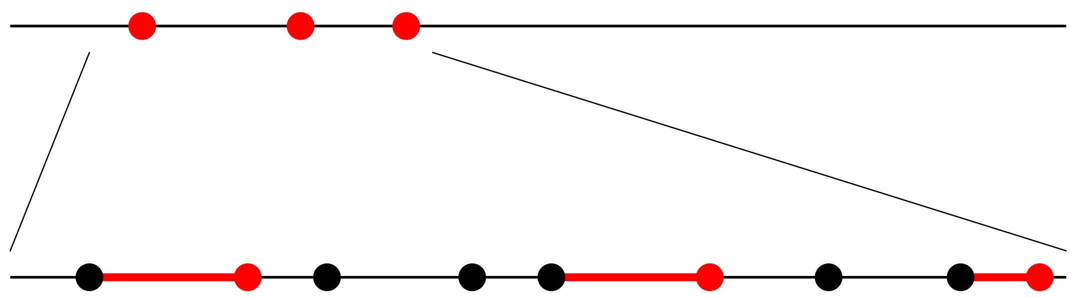

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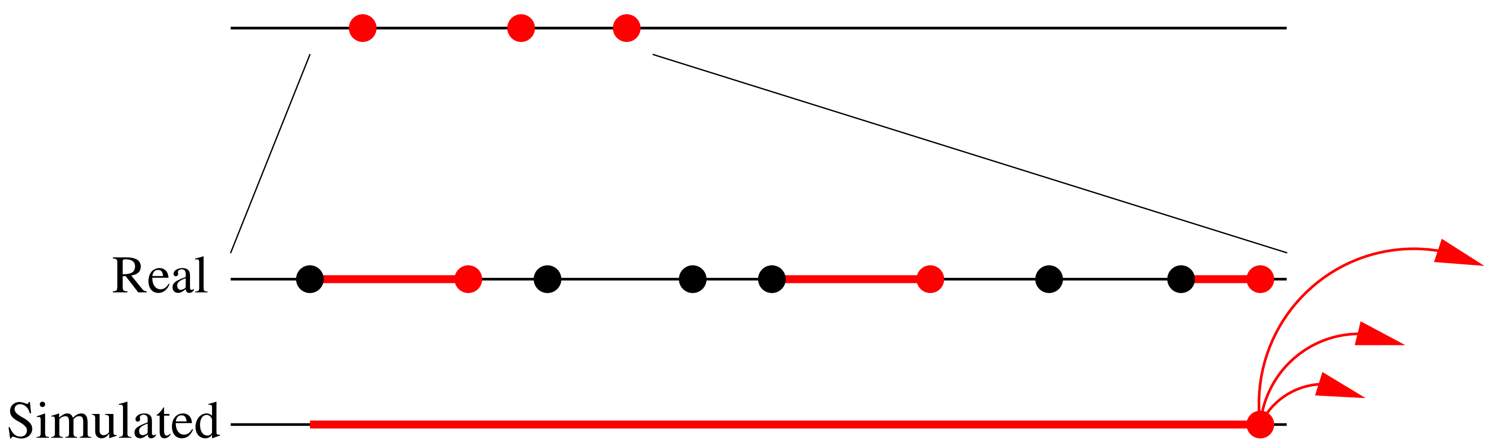
Our Approach: Overlay Topology

- When building overlay network, *simulate ownership of contiguous fraction* $\Theta(\frac{c_v \log n}{n})$ of ID space



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- But clustering of IDs $\Rightarrow$ real owner is nearby in ID space $\Rightarrow$ can complete route in $O(1)$ more hops using successor links

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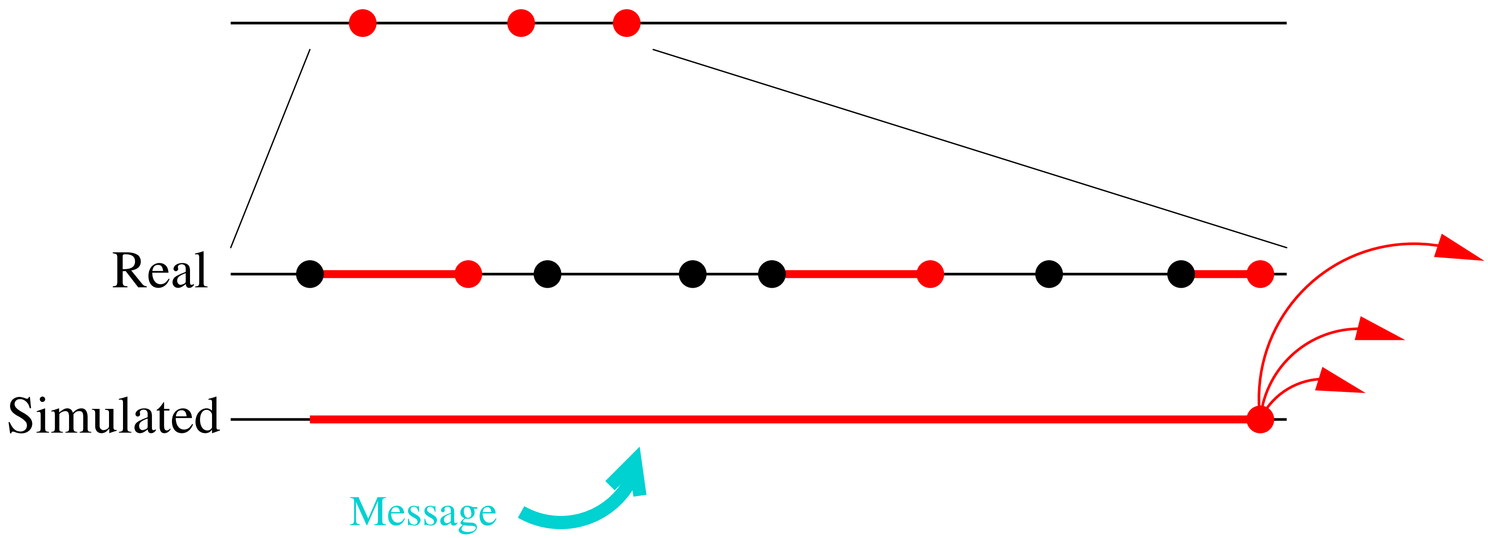

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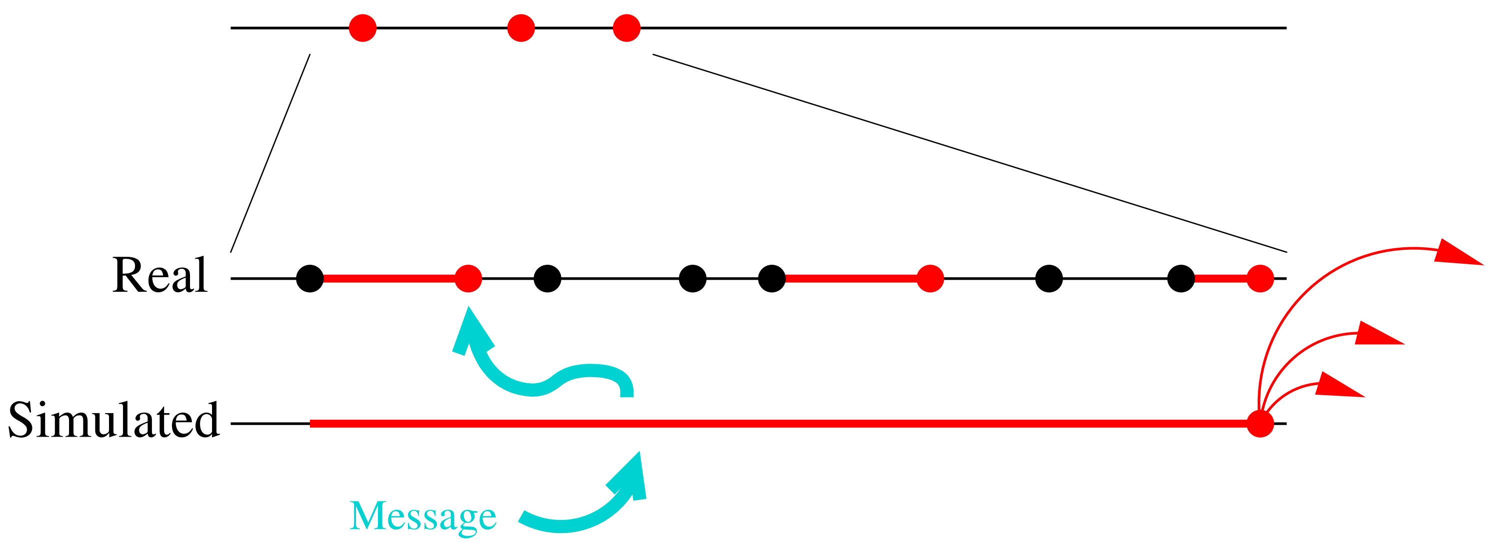

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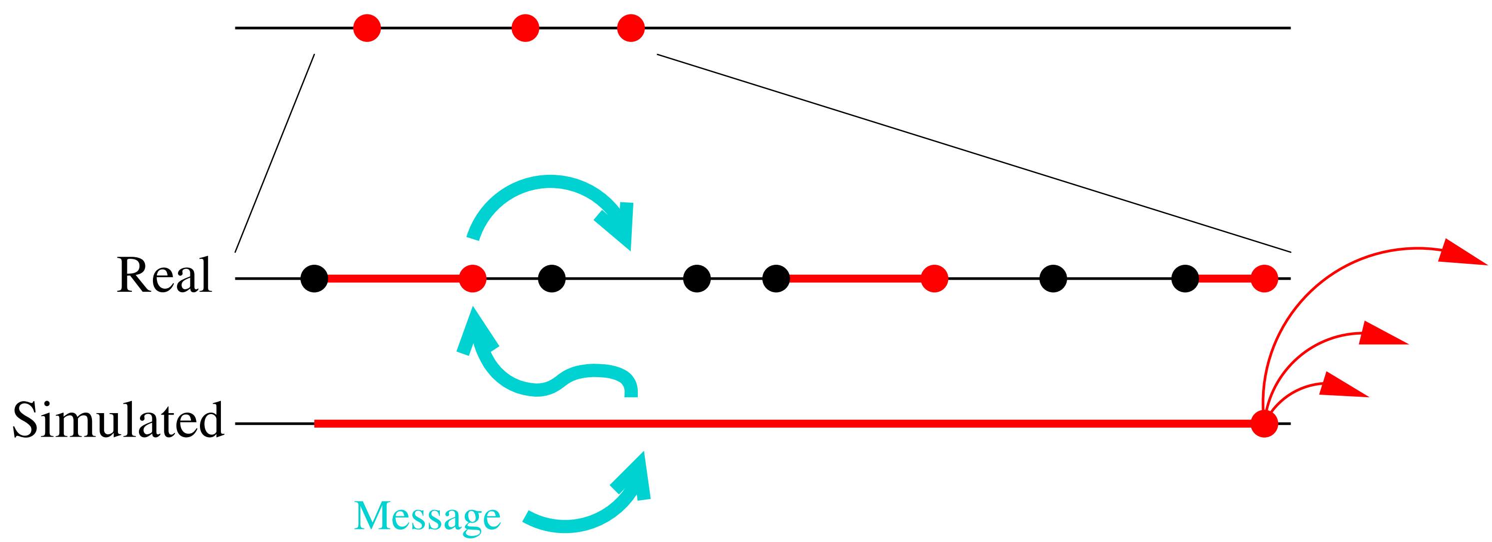

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Our Approach: Properties

- Works for *any* ring-based overlay topology;
  compared to single-ID case,
  - Node outdegree increases by at most a constant factor
  - Route length increases by at most an additive constant
- **Goals 1 & 2**: Load balance & handling heterogeneity
  - Achieves maximum share of $1 + \varepsilon$ for any $\varepsilon > 0$ and any capacity distribution
  - Tradeoff: outdegree depends on ε

Max Share Proof

Lemma 1 *If node v has at least one ID in the ring and $\alpha = \Theta(\log n)$, then (1) v has between $\alpha c_v/(\gamma_c\gamma_u) - O(1)$ and $\alpha c_v\gamma_c\gamma_u + O(1)$ IDs w.h.p., and (2) v has at least $\gamma_d\alpha(n) - O(1)$ IDs w.h.p.*

Proof: (1) Note that, due to the estimaton error parameters, the factor γ_c lazy update of $\tilde{c}_v$, and the factor 2 lazy update of $\tilde{n}$, we always have $\tilde{c}_v$ within a factor $\gamma_c\gamma_u$ of c_v and $\tilde{n}$ within a factor $2\gamma_n$ of n w.h.p. Thus, for some constant k, the number of IDs that v chooses is at most $\lfloor 0.5 + \tilde{c}_v\alpha(\tilde{n})\rfloor \leq \tilde{c}_v\alpha(\tilde{n}) + O(1) \leq \gamma_c\gamma_u c_v k\log(2\gamma_n n) + O(1) \leq \gamma_c\gamma_u\alpha(n) + O(1)$. The lower bound follows similarly, noting that we are not concerned with nodes that have been discarded. (2) Similarly, if v has decided to stay in the ring, we must have $\tilde{c}_v \geq \gamma_d$ and the bound follows by the above technique. ■

We now break the ring into *frames* of length equal to the smallest spacing parameter s_{min} used by any node. The following lemma implies that $s_{min} \geq 1/(2\gamma_n n)$ w.h.p.

Lemma 2 *Let $\beta = (1 - \gamma_c\gamma_u\gamma_d)/(\gamma_c\gamma_u)$. When $\alpha \geq \frac{8\gamma_n}{\beta\varepsilon^2}\ln n$, each frame contains at least $(1-\varepsilon)\beta\alpha n s_{min} - O(1)$ IDs w.h.p. for any $\varepsilon > 0$.*

Proof: Assume that no node has more than one ID in any frame; if this is not the case, we can break the high-capacity nodes for which it is false into multiple "virtual nodes" without disturbing the rest of the proof.

Consider any particular frame f. Let X_v be the indicator variable for the event that node v chooses an ID in f and let $X = \sum_v X$. We wish to lower-bound X. Suppose v chooses m_v points. Since f covers a fraction s_{min} of the ID space, we have $E[X_v] = m_v s_{min}$. By Lemma **??**, $m_v \geq \alpha c_v/(\gamma_c\gamma_u) - O(1)$ for nodes R in the ring. Thus,

$$
\begin{aligned}
\mathrm{E}[X] &= \sum_{v\in R}\mathrm{E}[X_v] \\
&\geq \sum_{v\in R} s_{min}\left(\alpha c_v/(\gamma_c\gamma_u) - O(1)\right) \quad \text{(Lemma ??)} \\
&\geq -O(1) + \sum_{v\in R}\frac{s_{min}\alpha c_v}{\gamma_c\gamma_u} \\
&= -O(1) + \frac{s_{min}\alpha}{\gamma_c\gamma_u}\sum_{v\in R} c_v \\
&\geq -O(1) + \frac{s_{min}\alpha}{\gamma_c\gamma_u}\cdot(1-\gamma_c\gamma_u\gamma_d)n \quad \text{(Claim ??)} \\
&= \beta\alpha n s_{min} - O(1),
\end{aligned}
$$

with β defined as in the lemma statement. (Note that although Claim **??** was stated in the context of Chord, it applies to our partitioning scheme without modification.) A Chernoff bound tells us that

$$
\begin{aligned}
\Pr[X < (1-\varepsilon)\mathrm{E}[X]] &< e^{-(\beta\alpha n s_{min} - O(1))\varepsilon^2/2} \\
&= O(e^{-\beta\alpha n s_{min}\varepsilon^2/2}) \\
&< e^{-\beta\alpha\varepsilon^2/(4\gamma_n)} \quad \text{(Lemma ??)} \\
&= O(n^{-2})
\end{aligned}
$$

when $\alpha \geq \frac{8\gamma_n}{\beta\varepsilon^2}\ln n$. Again by Lemma **??**, there are at $\leq 2\gamma_d n$ frames, so the lemma follows from a union bound over them. ■

Proof: (Of Theorem **??**) If node v is discarded, its share is 0, so we need only consider nodes in the ring. Such a node v chooses one ID in each of $m \leq \alpha c_v\gamma_c\gamma_u + O(1)$ frames (Lemma **??**).

We first fix the nodes' choices of the frames in which they place their IDs. Let $X_1,\ldots,X_m$ be the fraction of the ID space owned by each of node v's IDs. The randomness in the X_is is over the intra-frame positions of the nodes' IDs, which are chosen independently and uniformly at random. By Lemma **??**, we may assume that each frame has at least one ID. Thus, the interval assigned to the ith ID may span at most one frame boundary, so X_i depends only on the locations of the IDs in its frame and in the counterclockwise preceding frame. Thus, the odd-indexed X_is are mutually independent, as are the even-indexed X_is. We will bound the share of these two groups in the same way, one at a time. Consider first the odd-indexed X_is.

Break each frame into d *buckets* of equal size; we'll pick d later. A bucket is *occupied* when some node other than v chooses an ID inside it, and is *empty* otherwise. To analyze the node v's share of the ID space, we'll count the number of empty buckets counterclockwise-following v's chosen IDs. Define an infinite sequence of random variables Y_j, each of which will be the indicator variable for the event that a particular bucket is occupied. Y_1 will correspond to the bucket counterclockwise-following v's first odd-indexed ID. Suppose Y_j corresponds to the kth bucket following v's ℓth ID. Then we have two cases. (1) If $Y_j = 0$, Y_{j+1} corresponds to the next bucket following the same ID. (2) Otherwise, Y_{j+1} corresponds to the first bucket following the next odd-indexed ID, i.e. the $(\ell+2)$th one. If $m/2 < \ell + 2$ then we simply set $Y_{j+1} = 1$. Thus, the number of zeros in the sequence of Y_j's is the number of buckets entirely owned by v's $m/2$ odd-indexed IDs.

With the goal of upper-bounding the number of zeros, we first deal with dependence among the Y_js. By Lemma **??** we may assume that each frame has at least $r = (1-\varepsilon)\beta s_{min} n\alpha(n) - O(1)$ IDs for sufficiently large α. View $Y_1, Y_2, \ldots$ as a process. If $Y_{j-1} = 1$, then we are in Case (2) and Y_j corresponds to a frame independent of those of $Y_1,\ldots,Y_{j-1}$, so there are at least r IDs distributed u.a.r. in the frame which may occupy Y_j's bucket. If we are in Case (1) then Y_j's bucket is in the same frame as that of Y_{j-1}, which implies that some of the buckets in that frame are empty, in which case there are at least r IDs distributed u.a.r. in a *subset* of the frame including Y_j's bucket. This discussion implies that, regardless of the history of the Y_js, the probability that $Y_j = 1$ is at least $1-(1-1/d)^r$. Formally, we define another sequence of variables Z_j which are independent Poisson trials with success probability p to be picked below. For any indeces $j_1,\ldots,j_k$, we have

$$
\begin{aligned}
\Pr[Y_{j_1} = \cdots = Y_{j_k} = 1] &= \prod_{\ell=1}^{k}\Pr[Y_{j_\ell} = 1 | Y_{j_1} = \cdots = Y_{j_{\ell-1}} = 1] \\
&\geq \prod_{\ell=1}^{k}\left(1 - \left(1 - \frac{1}{d}\right)^r\right) \\
&\geq \left(1 - e^{-r/d}\right)^k \\
&= \Pr[Z_{j_1} = \cdots = Z_{j_k} = 1]
\end{aligned}
$$

where we have chosen the success probability for the Z_js to be $p = 1 - e^{-r/d}$. This implies that an upper bound the number of 0's in the independent Z_j sequence is also an upper bound the number of 0's in the dependent Y_k sequence, a fact which we use next.

If we see $m/2$ ones in the first x Y_js, then by the definition of the sequence, we have seen all the zeros, of which there are at most $x - m/2$. Thus node v will own at most $x - m/2$ complete buckets, plus $2\cdot m/2$ partial buckets (one at each end of the $m/2$ contiguous sequences of complete buckets), for a total of at most $x + m/2$ buckets due to its $m/2$ odd-numbered IDs. We now show that we see the required $m/2$ succeses w.h.p. when $x = \frac{m}{2p(1-\delta)}$. Let P be the number of 1's in the first x Y_js, and let P' be the corresponding value for the Z_js. By the above discussion we have $\Pr[P < m/2] \leq \Pr[P' < m/2]$, and $\mathrm{E}[P'] = xp = \frac{m}{2(1-\delta)}$, so

$$
\begin{aligned}
\Pr[P < m/2] &\leq \Pr[P' < m/2] \\
&= \Pr[P' < (1-\delta)\cdot\frac{m}{2(1-\delta)}] \\
&\leq e^{-\frac{m\delta^2}{4(1-\delta)}} \quad \text{(Chernoff bound)} \\
&\leq O(e^{-\frac{\gamma_d\alpha\delta^2}{4(1-\delta)}}) \quad \text{(Lemma ?? part (2))} \\
&= O(n^{-2})
\end{aligned}
$$

when $\alpha \geq \frac{8(1-\delta)}{\gamma_d\delta^2}\ln n$. In this case, counting now both odd- and even-indexed points, node v owns at most $m + \frac{m}{p(1-\delta)}$ buckets, each of size s_{min}/d. Normalizing by v's fair share c_v/n, we have

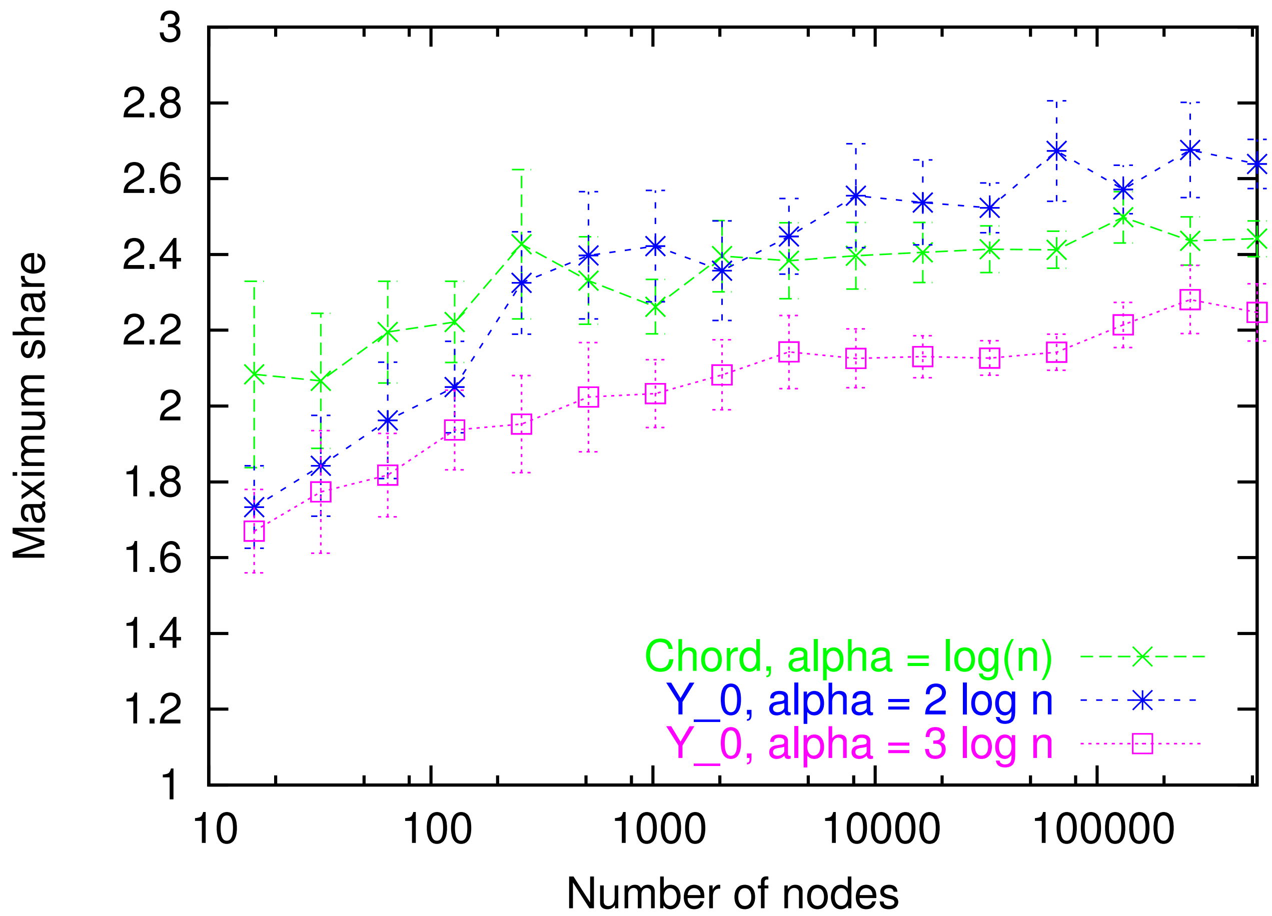
$$
\mathrm{share}(v) \leq \frac{1}{c_v/n}\cdot\left(\frac{m s_{min}}{d} + \frac{m s_{min}}{dp(1-\delta)}\right).
$$

Recall that d is arbitrary. Taking the limit as $d \to \infty$, we have $dp \to r = (1-\varepsilon)\beta s_{min} n\alpha(n) - O(1)$ so

$$
\begin{aligned}
\mathrm{share}(v) &\leq \frac{1}{c_v/n}\cdot\frac{m s_{min}}{(1-\delta)((1-\varepsilon)\beta s_{min} n\alpha(n) - O(1))} \\
&\leq \frac{1}{c_v}\cdot\frac{m}{(1-\delta)(1-\varepsilon)(1-\varepsilon')\beta\alpha(n)} \\
&\leq \frac{1}{c_v}\cdot\frac{\alpha(n)c_v\gamma_c\gamma_u + O(1)}{(1-\delta)(1-\varepsilon)(1-\varepsilon')\beta\alpha(n)} \quad \text{(Lemma ?? part (1))} \\
&\leq \frac{(1+\varepsilon'')(\gamma_c\gamma_u)^2}{(1-\delta)(1-\varepsilon)(1-\gamma_c\gamma_u\gamma_d)}
\end{aligned}
$$

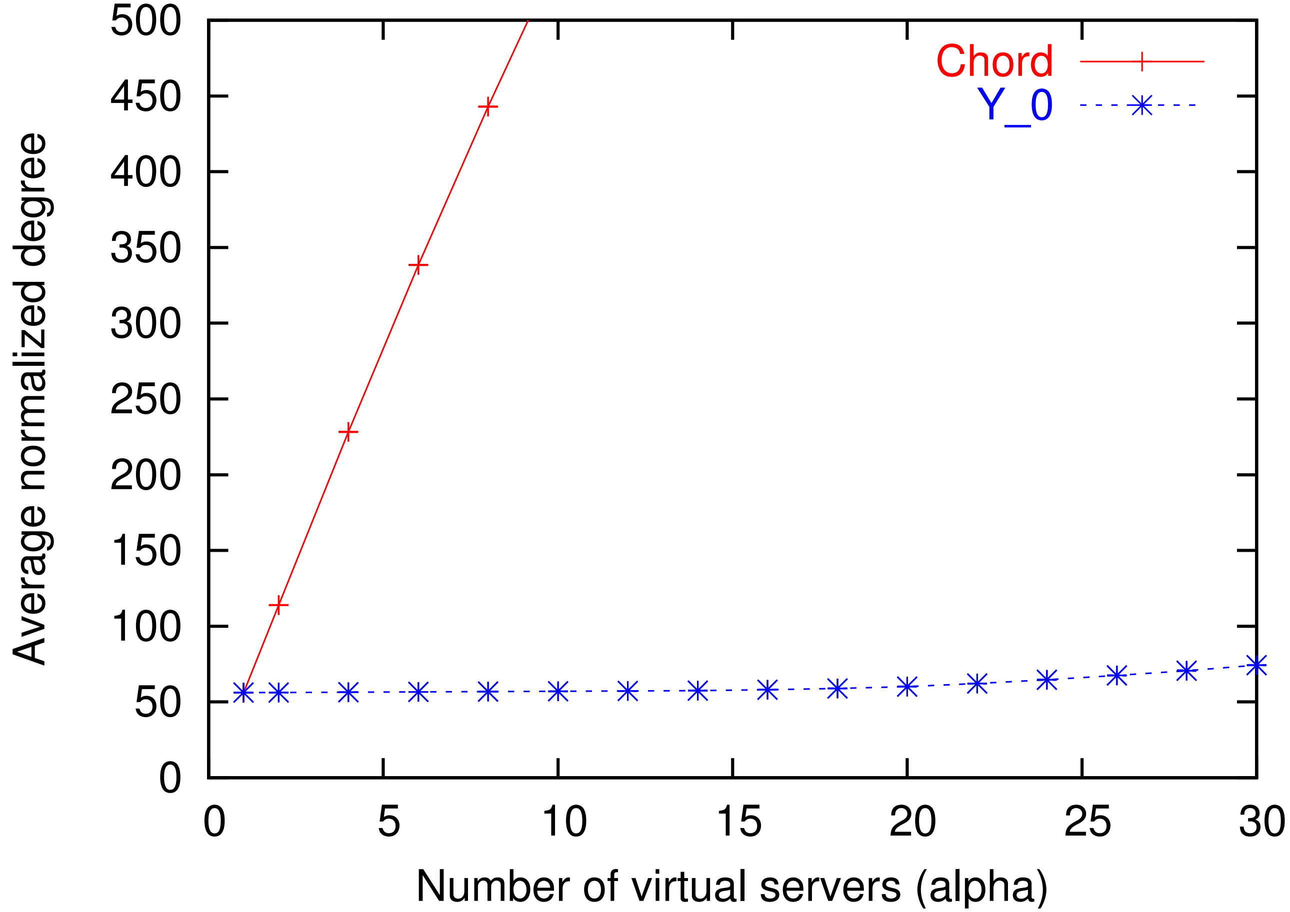
with probability $1 - O(n^{-2})$ for any $\varepsilon', \varepsilon'' > 0$ and sufficiently large n, so by a union bound, this is true of all nodes w.h.p. Finally, we require that α is the maximum of the requirement given above and that of Lemma **??**; setting $\delta = \varepsilon$ for convenience of presentation, we have $\max\{\frac{8(1-\varepsilon)\ln n}{\gamma_d\varepsilon^2}, \frac{8\gamma_n\gamma_c\gamma_u\ln n}{(1-\gamma_c\gamma_u\gamma_d)\varepsilon^2}\} \leq \frac{8\gamma_n\gamma_c\gamma_u\ln n}{(1-\gamma_c\gamma_u\gamma_d)\gamma_d\varepsilon^2}$, as required by the theorem. ■

Simulation: Maximum share



- **Parameter:** α = number of virtual servers per unit capacity
- Homogeneous capacities shown here
- Chord with $\alpha = 1$ increases to maximum share ≈ 13.7.

Simulation: Degree

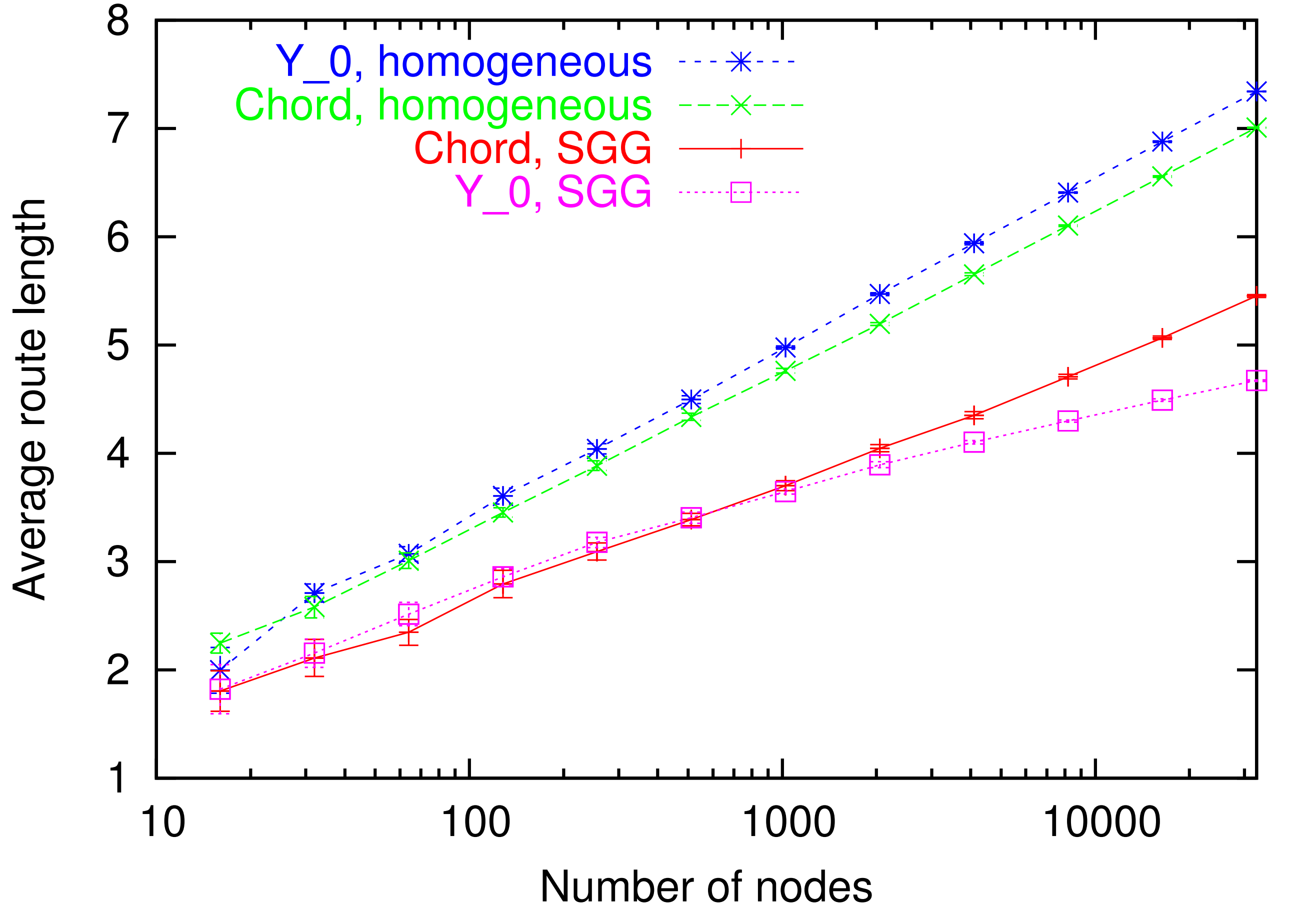


Degree of a node = number of links to other nodes

Exploit Heterogeneity (Goal 3)

- Even high-capacity nodes have a single set of overlay links
- Make use of unused capacity: pick denser set of links
- In Chord with $\alpha = 1$: $\Theta(c_v \log n)$ total outlinks
  - $\Theta(\log n)$ links in $\Theta(c_v)$ finger tables (one per virtual server)
- In our scheme: $\Theta(c_v \log n)$ total outlinks
  - ... all in one dense finger table
  - More structured topology $\Rightarrow$ reduced route length

Simulation: Effect of heterogeneity



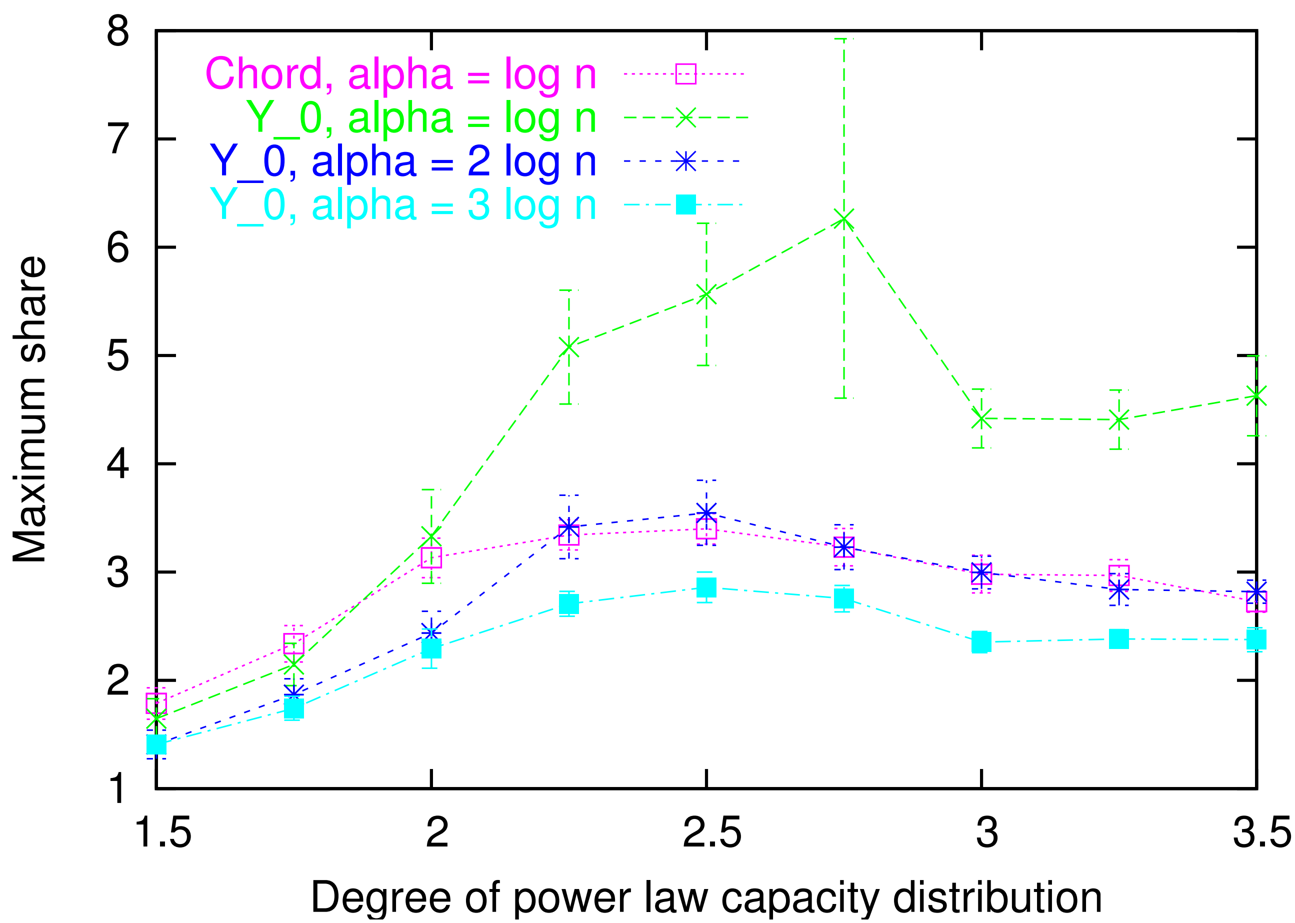
- SGG capacity distribution from real Gnutella hosts
- *Asymptotic* route lengths compared to homogeneous case
  Chord: $\leq$ 23% shorter Y_0: $\geq$ 55% shorter

Conclusion

- **Advantages**
  - Simple way to achieve good load balance at low cost
  - Compatible with any ring-based overlay
  - Adds flexibility in neighbor selection to any overlay
  - Takes advantage of heterogeneity to reduce route length
- **Disadvantages**
  - Some additional overhead, especially when particularly good balance desired
  - Will incur additional load movement when number of nodes or average capacity changes by a constant factor
- **Question:** where else does heterogeneity help distributed systems?

Backup slides

Simulation: Max Share vs. Capacity Distribution



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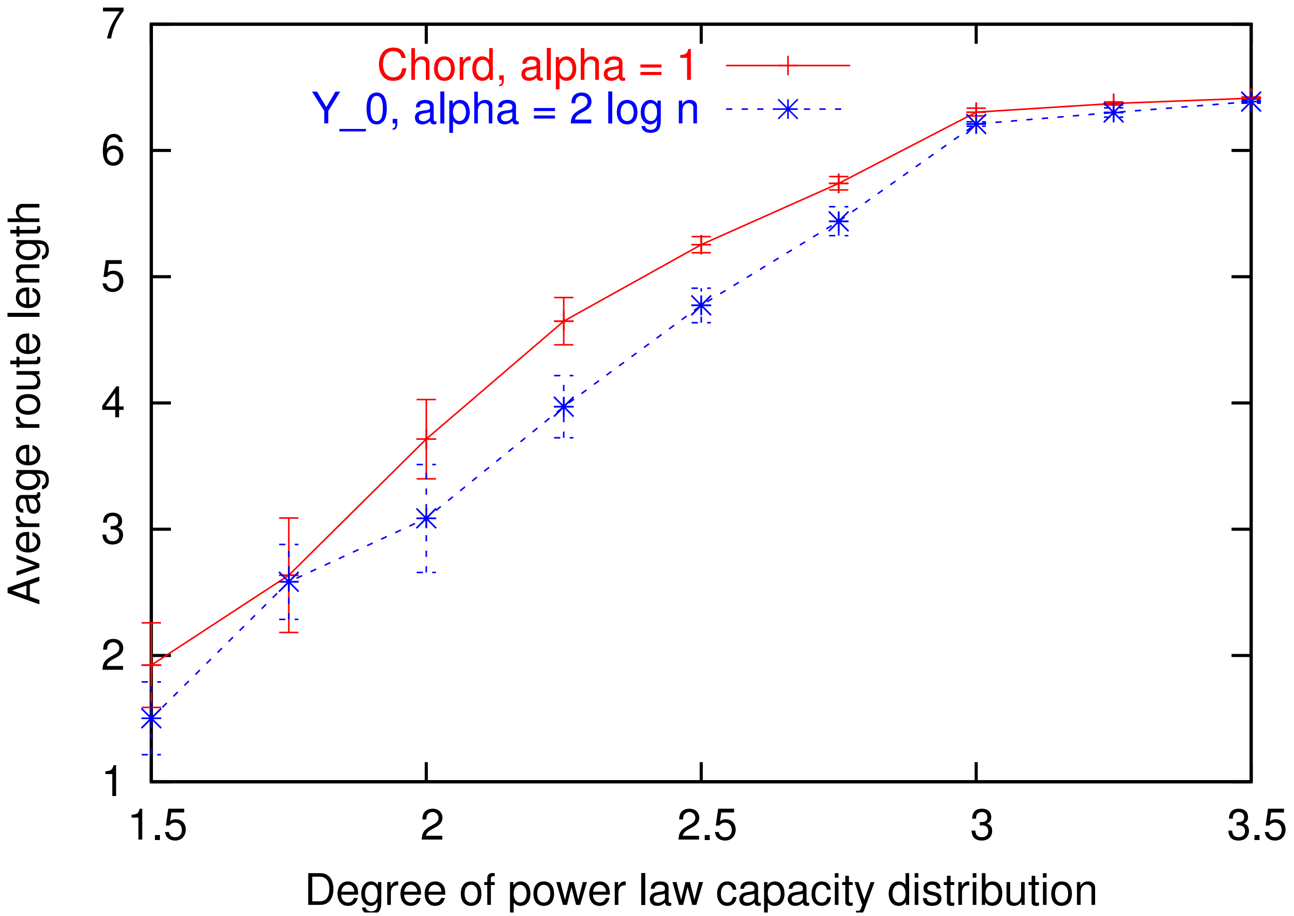

Route length vs. capacity distribution in a 16,384-node system.